ON THE ORIGIN OF OUTER BELT PROTONS

Part I

by

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Abstract

The variations in the energy spectra with pitch angle and L of the relatively stable 0.1 to 5 Mev protons in the outer radiation belt have been found to be in good agreement with the results of a model that permits rapid motion of the protons in L space. In this model the third adiabatic invariant of the protons is violated but not the first two adiabatic invariants. The variations in flux with L are found to indicate an external source and are discussed qualitatively.

Introduction

In the L = 2 to 5 range in the outer radiation belt, Davis, Hoffman, and Williamson [1964] find that the spectra of the relatively stable 0.1 to 5 Mev protons show smooth but large variations with L and equatorial pitch angle, α_o. Protons near the earth and at α_o near 90° are more energetic than those at larger L and at smaller α_o. The spectra are well represented by ℓ^{-E/E_o}; E_o varies by about a factor of 10 with L and by a factor of 2 with α_o. In this study a model is proposed for the explanation of these spectral variations.

Kellogg [1959] first suggested that the radiation belt might be formed through magnetic disturbances in which the third adiabatic invariant of trapped particles is violated without violating the first and second invariants. Violation of the third invariant allows motion in L space. As particles move closer to the earth they tend to gain energy with the maintenance of the first invariant since, for example, E/B is a constant for 90° pitch angles. So, this process can introduce acceleration of protons. Kellogg's suggestion has been adopted for this study although the mechanism for motion in L space is unspecified. It has further been assumed that motion in L-space is rapid compared with atmospheric loss and scattering processes except very near the earth and that the geomagnetic field is sufficiently well represented by a dipole.

Energy and Angle Variations

If the first and second adiabatic invariants of trapped particles are maintained during motion in L space, changes in both the energy and equatorial pitch angle can be calculated. The first invariant is

$$\mu = \frac{E \sin^2 \alpha_0}{B_0} = \frac{E\, L^3 \sin^2 \alpha_0}{.312} \qquad (1)$$

where B_o is the equatorial magnetic field. The second invariant is

$$J = m \oint v \cos \alpha \; d\, S \qquad (2)$$

where m is the mass, v the velocity, α the local pitch angle and S is along the guiding center. The integration is over a complete north-south oscillation. For a dipole magnetic field, equation (2) reduces to

$$J = v\, L\, F\, (\alpha_0)$$

$$F(\alpha_0) = 4 m r_e \int_0^{\lambda_m} \sqrt{\left[1 - \frac{\sin^2 \alpha_0 (4 - 3 \cos^2 \lambda)^{\frac{1}{2}}}{\cos^6 \lambda}\right] (4 - 3 \cos^2 \lambda)} \; \cos\lambda \; d\lambda \qquad (3)$$

where r_e is the radius of the earth, λ is the latitude, and λ_m is the mirror latitude. λ_m is given by

$$1 - \frac{\sin^2 \alpha_0 (4 - 3 \cos^2 \lambda_m)^{\frac{1}{2}}}{\cos^6 \lambda_m} = 0$$

and depends only on α_0.

Since μ and J are constants, equation (1) may be divided by the square of equation (3) to give

$$L \left[\frac{\sin \alpha_0}{F(\alpha_0)} \right]^2 = \text{constant.} \tag{4}$$

From this, the changes in α_0 with L can be evaluated; some results are shown in Figure 1. Two features of these results are worthy of note: (1) As Davis and Chang [1962] have indicated, particles diffusing inwards assume flatter helices; (2) Changes in α_0 with L are independent of energy for non-relativistic particles.

These changes in α_0 with L and equation (1) may be used to find the variation in energy with L and α_0. Results are shown in Figure 2 for protons having α_0 values at L = 7 as indicated on the curves. Energies are relative to energies at L = 7.

Discussion of the Observed Spectra

For the proposed model, the spectra of protons that one would expect at some L and α_0 depends on assumptions about the location and nature of the source and the energy dependence of motion in L space. The source is assumed to be at a single L and to consist of a single spectrum of protons. The result of the superposition of sources at different L and of different spectra can be obtained from the results of the above assumption. The energy dependence of motion in L space has been examined for two processes where the third invariant only is violated. When the violation mechanism involves electric fields, the velocity of L space motion is proportional to the vector product of the electric and the magnetic fields and does not depend on particle energy. Another process that produces violation of only the third invariant depends on asymmetric distortions of the geomagnetic field such as occur with sudden commencements and sudden impulses [Parker, 1960]. Motion in L space for this process depends on the guiding center of particles following magnetic field lines during rapid changes in the field and is also independent of energy. Thus, the assumption that motion in L space, when only the third invariant is violated, is independent of energy is certainly valid for known processes.

With thé above L motion results, source assumptions, and the energy transformations of the previous section, changes in spectra for motion in L space are readily obtained.

The energy transformation is given by

$$E\, L^3 \sin^2 \alpha_{oL} = E_S\, L_S^3 \sin^2 \alpha_{os} \tag{1A}$$

where the subscript s refers to the source. According to equation (1A) a power law source spectrum remains power law with the same exponent after L space motion. That is,

$$\frac{d\,E_s}{{E_s}^a} \rightarrow \frac{d\,E}{E^a} \,.$$

If the source spectrum has an exponential form, the transformation is given by

$$\ell^{-E_s/E_{os}}\, d\,E_s \rightarrow \ell^{-\frac{E\,L^3\,\sin^2\alpha_o\,L}{E_{os}\,{L_s}^3\,\sin^2\alpha_{os}}}\, d\,E \,.$$

From this it can be seen that an exponential source remains exponential after L space motion and that E_o changes in the same way with L and α_o as has been calculated for a single particle in the previous section. (i.e., the E_o transformation is given by Equation (1A)).

These two predictions of the model may be compared with experiment. The first prediction, that the spectra retains its exponential form, is in agreement with experiment. To test the second prediction, measured E_o [Davis et al., 1964] have been plotted in Figure 3 as a function of L with appropriate changes in α_o with L.* The labels on the curves refer to α_o values at L = 7. The dashed curves in Figure 3 are taken from

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*We would like to thank Drs. Davis, Hoffman and Williamson for making their data available to us before publication.

Figure 2 for corresponding changes in E with L and α_0. This comparison is only between the slopes of the curves (i.e., between relative changes in E_0 with L and α_0). The changes in E_0 with L show good agreement between the model and experimental results. The experimental results also give the same trend as the model in the change in the slopes of the curves with α_0.

If the dashed curves in Figure 3 are extended, they intersect near L = 10. This intersection is where the spectrum is independent of α_0 and thus gives a source location with the simplest assumptions about the source.

In the above discussion the data has been found compatible with a source at a single L value that consisted of a single exponential spectrum. It is, however, recognized that the data is also compatible (especially if the motions in L space are due to a diffusion type mechanism) with (a) the source location being near L = 10 but spread over a range of L values; (b) the source spectrum not being exponential but the time and space average of the source spectrum being exponential.

The Measured Velocity Distribution

A great virtue of the measurements by Davis, Hoffman and Williamson is that the velocity distribution function f can be obtained from them, and since f(E, α_0, L) was found not to change appreciably with time, the lack of simultaneity in the measurements seems unimportant.

In our calculation thus far, only the spectra of particles has been compared with the model. We can try to study particle fluxes during the

L drift by using the Liouville Theorem. With motion in L space, the fluxes may obey Liouville's Theorem or not obey Liouville's Theorem if losses or processes which change μ or I or diffusion-like processes are involved on a macroscopic scale.

The usual form of Liouville's Theorem used, for example, in cosmic ray problems states that the density of particles in phase space, f, is constant along the particle's trajectory.

$$f(v, \alpha) = \frac{dN}{dA\ dt\ p^2\ dp\ d\Omega} \quad . \tag{5}$$

But, the pitch angle distribution in protons/cm^2-sec-ster-Mev measured by Davis, Hoffman and Williamson is

$$j(E, \alpha) = \frac{dN}{dA\ dt\ dE\ d\Omega} \quad . \tag{6}$$

This gives

$$f(v, \alpha) = \frac{j(E, \alpha)\ dE}{p^2\ v\ dp} \tag{7}$$

or

$$f(E, \alpha) = \frac{1}{2\ m\ E}\ j(E, \alpha) \quad . \tag{8}$$

This shows the simple relationship between the measured fluxes and the velocity distribution function. In studying the constancy of the density in phase space $f(E, \alpha)$ we must follow the particle's motion along a dynamic trajectory. Our model of the drift process has μ and I constants of the motion. Therefore, we test the Liouville Theorem by asking

$$f(E_1, \alpha_1, L_1) \stackrel{?}{=} f(E_2, \alpha_2, L_2) \tag{9}$$

where E_1 and α_1 are related to E_2 and α_2 by equations (1) and (4). That is

$$f[E_1(\mu_1, I_1), \alpha_1(\mu_1, I_1), L_1] \stackrel{?}{=} f[E_2(\mu_1, I_1), \alpha_2(\mu_1, I_1), L_2] \quad . \tag{10}$$

The values of f were computed as functions of L for many pairs of the values of μ and J, and Figure 4 shows f plotted against L for fixed μ and J. This shows f varying considerably with L. Therefore, the particles do not obey the Liouville Theorem during their L drifting motion. It is seen that $(\partial f/\partial L)_{\mu J}$ is always positive, suggesting that the particle source is at large L, the particles diffusing inwards and loss processes reducing f further in. The small slopes of Figure 4 at the larger L values implies that loss processes are probably relatively unimportant there, the slope probably being due to diffusion of particles away from the source at the outer boundary. The much larger slopes at the lower L values imply that loss processes are relatively important in this region. If loss processes are important anywhere in the L drift motion certain restrictions must be placed on them. They must not change the observed shape of the energy spectra. Therefore, the loss process must be essentially energy independent in the range of the observations. This limits the possible processes considerably because many loss processes such as dE/dx and charge exchange are strongly energy dependent.

If, as is suggested here, the source of the outer belt protons is at the magnetopause we can infer some of their properties from our model. Because the variation of equatorial pitch angle with L is slow (see Figure 1) the angular distribution will resemble that measured by Davis and Williamson [1963, Figure 4] in the outer belt. The source distribution may be a little wider than this. The time averaged energy spectrum for a magnetopause source near L = 10 should be exponential with a characteristic energy, E_o, of about 15 Kev.

Conclusions

In this study a rather simple model has been found to successfully explain spectral changes with L and α_o of the protons in the outer radiation belt measured by Davis, Hoffman and Williamson. The trends in the variations in the spectra and in comparison of fluxes with Liouville's Theorem both indicate that the source is near the edge of the magnetosphere. According to this model, the protons are moved within the outer radiation belt and accelerated by some mechanism that violates the third adiabatic invariant of charged particle motion without violating the first two invariants. The variation of flux with L suggests that diffusion or loss processes are relatively important especially at the lower L values.

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Figure Captions

Figure 1 - Variation in equatorial pitch angle with L when μ and J are constant.

Figure 2 - Relative variation in proton energy with L when μ and J are constant.

Figure 3 - Comparison between measured and predicted variations in E_o with L and α_o.

Figure 4 - Relative variation of j/E with L.